# Effect of a dipole moment on the wake potential of a dust grain in a flowing plasma

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The effect of a dipole moment on the wake potential of a dust grain in a collisionless plasma with a supersonic ion flow is studied. It is found that both the point charge and the dipole moment can be responsible for the oscillatory potential behind the dust. The dipole moment is dominant in forming the wake potential when the dipole moment $p$ becomes of the order of $|Q|\lambda_D$, where $Q$ is the dust charge and $\lambda_D$ is the Debye length.

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The theory of the wake potential has been proposed as a possible candidate for the formation of vertical arrangements and alignment of grains in the dust-plasma crystals [1–4]. The wake potential was experimentally confirmed to be responsible for the attraction of two dust grains in a plasma crystal [5]. The simplest theory is based on a test dust particle which is characterized by a point charge. However, the recent observation of the formation of plasma crystals of finite-size dust grains, both in experiments [6] and in simulations [7], prompted us to investigate the effect of the dipole moment on the formation of the wake potential. The appearance of a large dipole moment for a dielectric dust grain in a supersonic flow has been studied in the context of the charging process on the surface of the dust grain, but using an unscreened Coulomb potential neglecting the plasma (and therefore wake) effects [8].

In this Brief Report, we point out the effect of a dipole moment on the formation of the wake potential behind a dust grain in the presence of a flowing plasma. We do not attempt the difficult self-consistent calculation of the dust charge and dipole moment in an array of grains with wake potentials here. We instead consider the wake potential produced by a single dust grain which is characterized by a given charge $Q$ and a dipole moment $\mathbf{p}$. For simplicity, we do not consider the process of dust charging, thereby assuming that the test dust grain has a given (constant) charge and a dipole moment, for example those calculated without the wake potential effect, for a grain in various gases in Ref. [8]. Furthermore, the dust grain is assumed to be placed in the ion flow in a collisionless plasma which supports plasma collective modes. Our approach of treating the charge and dipole moment as parameters is necessary to form a basis for later self-consistent calculations which can avoid unphysical assumptions such as the neglect of the wake fields.

The potential of a distributed test charge in a plasma without ion flow may be written, retaining the monopole and dipole contributions, as

$$\phi=\frac{q}{r}\exp\left(-\frac{r}{\lambda_D}\right)+\frac{\mathbf{p}\cdot\mathbf{e}_r}{r^2}\left(1+\frac{r}{\lambda_D}\right)\exp\left(-\frac{r}{\lambda_D}\right), \tag{1}$$

where $\mathbf{p}$ is the dipole moment of a test charge $q$, $\mathbf{e}_r$ is a unit vector in the direction $\mathbf{r}$, $r=|\mathbf{r}|$, and $\lambda_D$ is the plasma Debye length.

As has been shown previously, the first term should be modified in a plasma with an ion flow in a way to produce a wake potential behind a point test charge [1–4]. Oscillations of the potential occur due to the excitation of an ion-acoustic wave standing in the flow downstream of the dust grain [1]. In this paper we show how the dipole moment will modify the wake potential behind a test dust grain in an ion flow. Let the dust grain be made up of distributed point charges $q_j$, so the charge density is

$$\rho_Q(\mathbf{r},t)=\sum_j q_j\delta(\mathbf{r}-\mathbf{r}_j-\mathbf{v}_jt)$$
$$=\frac{1}{V}\sum_j\sum_{\mathbf{k}} q_j\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_j-\mathbf{v}_jt)], \tag{2}$$

where $V$ is the volume of the system, and $\mathbf{r}_j$ and $\mathbf{v}_j$ are the location and the velocity of the distributed dust charges, respectively. We assume a small displacement $\Delta\mathbf{r}_j$ about the point $\mathbf{r}_0$ for each distributed charge, or $\mathbf{r}_j=\mathbf{r}_0+\Delta\mathbf{r}_j$, and set $\mathbf{v}_j=\mathbf{v}$. Expanding $\exp(-i\mathbf{k}\cdot\Delta\mathbf{r})\approx 1-i\mathbf{k}\cdot\Delta\mathbf{r}$, we obtain

$$\rho_Q(\mathbf{r},t)=\frac{1}{V}\sum_{\mathbf{k}} Q\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_0-\mathbf{v}t)]$$
$$-\frac{1}{V}\sum_{\mathbf{k}} i\mathbf{k}\cdot\mathbf{p}\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_0-\mathbf{v}t)], \tag{3}$$

where

$$Q=\sum_j q_j, \qquad \mathbf{p}=\sum_j \Delta\mathbf{r}_jq_j. \tag{4}$$

Following our previous procedure [4], we find the potential of the dust grain to be

$$\phi(\mathbf{r},t)=\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi}\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\phi(\mathbf{k},\omega), \tag{5}$$

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where Im $\omega > 0$ and

$$\phi(\mathbf{k},\omega) = \frac{4\pi}{Vk^2}\frac{i}{\omega - \mathbf{k}\cdot\mathbf{v}}\frac{\exp(-i\mathbf{k}\cdot\mathbf{r}_0)}{\varepsilon(\mathbf{k},\omega)}(Q - i\mathbf{k}\cdot\mathbf{p}). \quad (6)$$

We note that the dipole correction appears in such a way as to effectively modify the charge $Q$ in Eq. (6). The poles in Eq. (6) are from

$$\omega - \mathbf{k}\cdot\mathbf{v} = 0 \quad (7)$$

and

$$\varepsilon(\mathbf{k},\omega) = 0. \quad (8)$$

As in Refs. [1–4], we choose the dielectric response function for a plasma with cold flowing ions, and finite temperature electrons,

$$\varepsilon(\mathbf{k},\omega) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pi}^2}{(\omega - k_z v_0)^2}, \quad (9)$$

where $v_0$ is the ion flow velocity in the $z$ direction (we set $v_0 < 0$) and $\lambda_D$ in the case of a supersonic ion flow coincides with the electron Debye length (since ions cannot participate in the shielding process). The dielectric function may be conveniently written as

$$\frac{1}{\varepsilon(\mathbf{k},\omega)} = \frac{1}{\varepsilon^{(e)}(\mathbf{k},\omega)} + \frac{1}{\varepsilon^{(i)}(\mathbf{k},\omega)}, \quad (10)$$

where the electron and ion contributions are separated as

$$\frac{1}{\varepsilon^{(e)}(\mathbf{k},\omega)} = \frac{k^2\lambda_D^2}{1 + k^2\lambda_D^2} \quad (11)$$

and

$$\frac{1}{\varepsilon^{(i)}(\mathbf{k},\omega)} = \frac{k^2\lambda_D^2}{1 + k^2\lambda_D^2}\frac{\omega_k^2}{(\omega - k_z v_0)^2 - \omega_k^2}. \quad (12)$$

Here, we define

$$\omega_k^2 = \frac{k^2 C_s^2}{1 + k^2\lambda_D^2}, \quad (13)$$

with $C_s$ being the ion-acoustic speed. We note that the contribution to the dipole potential term in Eq. (5), due to the electron dielectric function (11), can be written, in the limit $V \to \infty$, as

$$\phi_{I0}^{P}(\mathbf{r},t) = 4\pi\int \frac{d^3k}{(2\pi)^3}\frac{\lambda_D^2}{1 + k^2\lambda_D^2}(-i\mathbf{k}\cdot\mathbf{p}) \times \exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{r}_0 - \mathbf{v}t)], \quad (14)$$

where the subscript $I0$ indicates the contribution from the pole given by Eq. (7), just with the electron term Eq. (11). Equation (14) may be evaluated for spherical symmetry assuming $\mathbf{r}_0 = \mathbf{v} = 0$ (i.e., for a stationary grain at the origin of the reference frame) and $d^3k = 2\pi k^2 \sin\theta d\theta dk$ as

$$\phi_{I0}^{P}(\mathbf{r},t) = \frac{\mathbf{p}\cdot\mathbf{e}_r}{r^2}\left(1 + \frac{r}{\lambda_D}\right)\exp\left(-\frac{r}{\lambda_D}\right). \quad (15)$$

On the other hand, for cylindrical symmetry assuming $\mathbf{r}_0 = \mathbf{e}_z z_0$, $\mathbf{v} = \mathbf{e}_z v$, and $d^3k = k_\perp dk_\perp dk_z d\Theta$ we obtain

$$\phi_{I0}^{P}(\rho,z,t) = \frac{p}{\rho\lambda_D}\frac{z - z_0 - vt}{|z - z_0 - vt|}\exp\left(-\frac{|z - z_0 - vt|}{\lambda_D}\right), \quad (16)$$

where $p$ is the $z$ component of $\mathbf{p}$, and the cylindrical coordinates $(\rho, z)$ are used. For an ion flow induced dipole moment, $\mathbf{p}$ is in the reverse direction to the ion flow $(p > 0)$. Equation (15) agrees with the second term of Eq. (1). As was done previously [4], all the pole contributions should be taken into account. We find that the potential given by Eq. (16) is canceled in the downstream direction by the part of the potential produced by the ion contribution, Eq. (12). Thus the total potential in the range $z_0 + v_0 t < z < z_0 + vt$ and $|z - z_0 - vt| > \rho(M^2 - 1)^{1/2}$ may be written as

$$\phi(\rho,z,t) = \frac{2M^2}{(M^2 - 1)^{3/2}}\int_0^{1/\lambda_D} dk_\perp (k_\perp\lambda_D)^2 J_0(k_\perp\rho) \times \left\{ Q \sin\left[\frac{k_\perp(z - z_0 - vt)}{\sqrt{M^2 - 1}}\right] - \frac{k_\perp p}{\sqrt{M^2 - 1}}\cos\left[\frac{k_\perp(z - z_0 - vt)}{\sqrt{M^2 - 1}}\right]\right\}, \quad (17)$$

where $J_0$ is the zeroth-order Bessel function of the first kind; the Mach number is defined as $M = |v - v_0|/C_s$. The near-field approximation, $k_\perp\rho < 1$, is given by, for $|z - z_0 - vt| > \lambda_D$,

$$\phi(z,t) = \frac{2/(1 - M^{-2})}{|z - z_0 - vt|} \times \left[\left(Q - \frac{p}{|z - z_0 - vt|}\right)\cos\left(\frac{z - z_0 - vt}{\lambda_D\sqrt{M^2 - 1}}\right) - \frac{p}{\lambda_D\sqrt{M^2 - 1}}\sin\left(\frac{|z - z_0 - vt|}{\lambda_D\sqrt{M^2 - 1}}\right)\right], \quad (18)$$

and for $|z - z_0 - vt| < \lambda_D$ by

$$\phi(z,t) = \frac{2}{M^2(1 - M^{-2})\lambda_D^2}[Q(z - z_0 - vt) - p]. \quad (19)$$

Equation (18) indicates that the dipole moment plays a major role in forming a wake potential if the magnitude of the dipole moment $p$ approaches the order of $|Q|\lambda_D\sqrt{M^2 - 1}$. We also see that around $z - z_0 - vt = -\lambda_D(M^2 - 1)^{1/2}$ the dipole moment will modify the potential structure in a way to distort the oscillatory character. A recent analysis of the dipole moment of a dust grain of radius $a$ placed in a super-

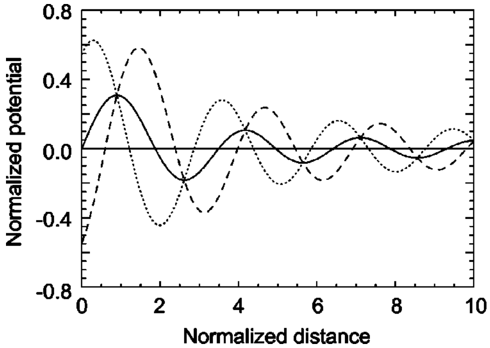


FIG. 1. The wake potential $\phi$, normalized by $2M^2|Q|/(M^2-1)^{3/2}\lambda_D$, at $\rho=0$, against distance downstream of the grain, normalized by $\lambda_D$. $M=1.1$ and $p=0$ (solid curve), $p=|Q|\lambda_D$ (dashed curve), and $p=-|Q|\lambda_D$ (dotted curve).

sonic plasma flow showed the magnitude of the induced dipole moment $p_{Fl}$ could be significant, with $p_{Fl}\sim a|Q|/2$, due to the assymmetric ion flow producing a strongly nonuniform charge distribution on the surface [8]. Thus the condition for the dipole moment to play a dominant role in forming the wake potential is $a\sim\lambda_D$. The dipole moment $\mathbf{p}_E$ induced by the sheath electric field (in the same direction as the ion flow, i.e., $\mathbf{p}_E\cdot\mathbf{e}_r<0$ or $p_\mathrm{E}<0$) can be only neglected in comparison with the dipole moment caused by the ion flow for a grain size $<40$ $\mu$m [8]. Note that the moment $\mathbf{p}_E$ is nonzero also for a conducting grain, in contrast to the case of the dipole moment induced by the plasma flow. Figure 1 shows the normalized wake potential for a negatively charged grain at $\rho=0$, plotted against the normalized distance downstream of the grain $|z-z_0-vt|/\lambda_D$, for $M=1.1$, and for $p/|Q|\lambda_D=0$ (no dipole moment), as well as for (extreme) cases $p_{Fl}/|Q|\lambda_D=1$ (ion flow induced dipole moment) and $p_E/|Q|\lambda_D=-1$ (electric field induced dipole moment). We see that the downstream potential maxima, where other dust grains may reside in stable equilibrium, are stronger and more distant from the original grain if the ion flow induced dipole moment is included, as is expected for this orientation of the moment, whereas the maxima are stronger and closer to the grain for the electric field induced dipole moment. These characteristics of the oppositely directed dipole moments may provide a way for experimental observations to distinguish between the mechanisms of creation of the dipole moment in the grains.

In conclusion, it has been shown that a charged dust grain with a dipole moment creates an oscillatory wake potential behind the grain, anologously to the monopole case. When the size of the dust grain becomes comparable to the Debye length, the dipole moment plays an essential role in the structure of the wake potential. The structure is found to depend on the directionality of the dipole moment. The potential of a dust grain given by Eq. (1) should be replaced by the potential given by Eq. (18) behind the particle in the presence of a supersonic ion flow when the ion-acoustic wave (standing in the dust grain reference frame) is generated within the Mach cone. If the wake potential contributes to the alignment of grains in a dust crystal, the separation of the grains in the ion flow direction will be larger due to the ion flow induced dipole moment. Finally, we note that the wake potential will in turn modify the process of dust charging (and inducing the dust dipole moment), making the fully self-consistent problem of dust charging and screening as well as the dust-dust interaction in the presence of an ion flow highly nonlinear.

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